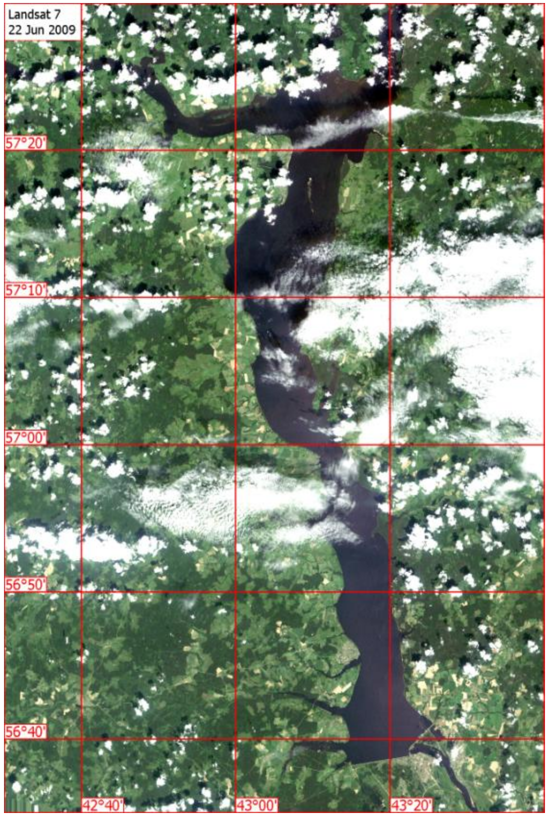

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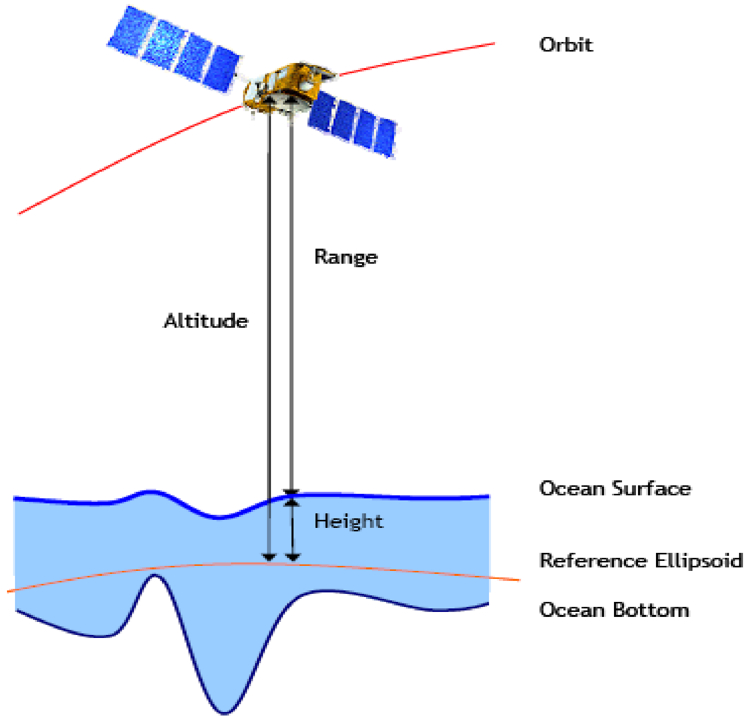
Satellite altimetry of inland water bodies. I. Physical principals and algorithms

The main problems to discuss in this lecture

- ✓ the general ideas and principals of classical satellite altimetry
- ✓ problems of usage of satellite altimetry in coastal zone, over land and inland water bodies
- ✓ the way to recover the problem – re-tracking, different methods of re-tracking

The general ideas and principals of classical satellite altimetry

Satellite altimetry



By parameters of the telemetric impulse of RA one can retrieve

- ✓ **Range is the height of the satellite over the water (land) surface**
- ✓ **Surface back-scatter coefficient (Sigma_0), which determines the wind speed**
- ✓ **Significant wave height (SWH)**

Altitude of the satellite over the reference ellipsoid with the equatorial radius 6378.1363 km and eccentricity 1/298257 (Alt)

Corrected Range = Range + Wet Troposphere Correction + Dry Troposphere Correction + Ionosphere Correction + Sea State Bias Correction

Sea Surface Height = Altitude - Corrected Range

Sea Level Anomaly = Sea Surface Height - Mean Sea Surface - Solid Earth Tide Height - Geocentric Ocean Tide Height - Pole Tide Height - Inverted Barometer Height Correction - HF Fluctuations of the Sea Surface Topography

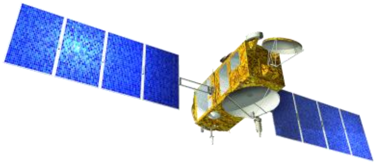
Parameters of the orbit of Jason –1,2

Semi-major axis	7,714.43 km
Eccentricity	0.000095
Inclination	66.04 deg
Argument of periapsis	90.0 deg
Inertial longitude of the ascending node	116.56 deg
Mean anomaly	253.13 deg

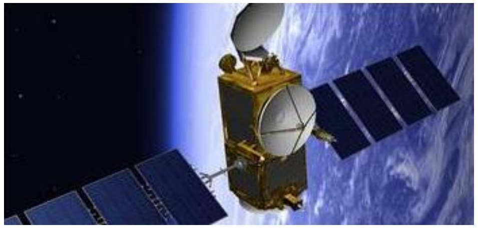
Reference (Equatorial) altitude	1336 km
Nodal period	6745.72 sec
Repeat period	9.9156 days
Number of revolutions within a cycle	127
Equatorial cross-track separation	315 km
Ground track control band	$\pm$ 1 km (at equator)
Acute angle at Equator crossings	39.5 deg
Longitude of Equator crossing of pass 1	99.9242 deg
Inertial nodal rate	-2.08 deg/day
Orbital speed	7.2 km/s
Ground track speed	5.8 km/s

Satellites TOPEX/Poseidon and Jason-1,2

Satellite TOPEX/Poseidon



Satellite Jason-1



Satellite Jason-2

- ✓ The accuracy of measuring the range (height of the satellite over the water (land) surface) is 2.9 cm, the accuracy of the sea surface height with respect to the reference ellipsoid is 3.1 cm.
- ✓ Repeat period is 9.9156 days
- ✓ The spatial resolution is 700 m, the declared accuracy is achieved with a spatial resolution of 5.8 km
- ✓ Data of TOPEX / Poseidon (from September 1992 to August 2002), Jason-1 (from January 2002 to 2012) and Jason-2 (June 2008 to present) together is an ongoing and the longest time series of measurements

Measurement error surface height, wave height and wind speed (Jason-1)

	OGDR		IGDR		GDR		Goal
	Spec.	Perf.	Spec.	Perf.	Spec.	Perf.	
Altimeter noise	2.5 cm (a)(c)(d)	TBD	1.7 cm (b)(c)(d)	TBD	1.7 cm (b)(c)(d)	TBD	1.5 cm (b)(c)(d)
Ionosphere	1 cm (e)(d)	TBD	0.5 cm (e)(d)	TBD	0.5 cm (e)(d)	TBD	0.5 cm (e)(d)
Sea State Bias	3.5 cm	TBD	2 cm	TBD	2 cm	TBD	1 cm
Dry troposphere	1 cm	TBD	0.7 cm	TBD	0.7 cm	TBD	0.7 cm
Wet Troposphere	1.2 cm	TBD	1.2 cm	TBD	1.2 cm	TBD	1 cm
Altimeter range : RSS	**5 cm**	**TBD**	**3 cm**	**TBD**	**3 cm**	**TBD**	**2.25 cm**
RMS Orbit (Radial component)	10 cm (h)	TBD	2.5 cm	TBD	1.5 cm	TBD	1 cm
SSH : Total RSS	**11.2 cm**	**TBD**	**3.9 cm**	**TBD**	**3.4 cm**	**TBD**	**2.5 cm**
Significant wave height	10% or 0.5 m (i)	TBD	10% or 0.4 m (i)	TBD	10% or 0.4 m (i)	TBD	5% or 0.25 m (i)
Wind speed	1.6 m/s	TBD	1.5 m/s	TBD	1.5 m/s	TBD	1.5 m/s
Sigma0 (absolute)	0.7 dB	TBD	0.7 dB	TBD	0.7 dB	TBD	0.5 dB
System drift	/	/	/	/	/	/	1mm/year (j)

CONDITIONS : 1 s. average, 2 m SWH, 11 dB Sigma0

(a) Combined Ku + C measurement
(b) Ku band after ground retracking
(c) Averaged over 1 sec
(d) Assuming 320 MHz C bandwidth
(e) Filtered over 100 Km
(f) Can also be expressed as 1% of H1/3
(g) After ground retracking (not applicable to Jason-2 but maintain for safe of consistency with Jason-1 OSDRs products)
(h) Real time DORIS onboard ephemeris
(i) Which ever is greater
(j) On global mean sea level, after calibration

Parameter	Validity conditions
range_numval_ku	10 ≤ x
range_rms_ku	0 ≤ x (mm) ≤ 200
altitude - range_ku	-130 000 ≤ x (mm) ≤ 100 000
model_dry_tropo_corr	-2 500 ≤ x (mm) ≤ - 1 900
rad_wet_tropo_corr	-500 ≤ x (mm) ≤ - 1
iono_corr_alt_ku	-400 ≤ x (mm) ≤ 40
sea_state_bias_ku	-500 ≤ x (mm) ≤ 0
ocean_tide_sol1	-5 000 ≤ x (mm) ≤ 5 000
solid_earth_tide	-1 000 ≤ x (mm) ≤ 1 000
pole_tide	-150 ≤ x (mm) ≤ 150
swh_ku	0 ≤ x (mm) ≤ 11 000
sig0_ku	7 ≤ x (dB) ≤ 30
wind_speed_alt	-0 ≤ x (m/s) ≤ 30
off_nadir_angle_wf_ku	-0.2 ≤ x (deg^2) ≤ 0.64
sig0_rms_ku	x (dB) ≤ 1
sig0_numval_ku	10 < x

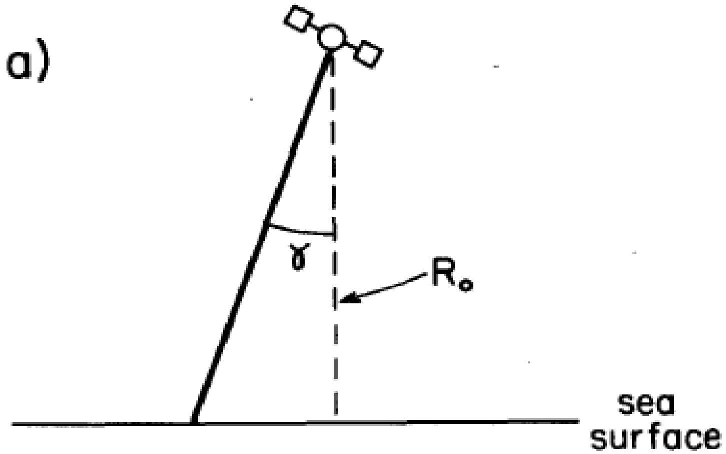
Table 12 : Recommended filtering criteria

Parameter	Validity conditions
Swh_c - swh_ku	-2 ≤ x (m) ≤ 2
Swh_rms_ku / [MAX(swh_ku,1)]$^{1/3}$	x < 18
Swh_rms_c / [MAX(swh_ku,1)]$^{1/3}$	x < 44
range_rms_ku / [MAX(swh_ku,1)]$^{1/3}$	x < 100
range_rms_c / [MAX(swh_ku,1)]$^{1/3}$	x < 170
sig0_rms_c	X (dB) < 0.26

Table 13 : Recommended additional empirical tests

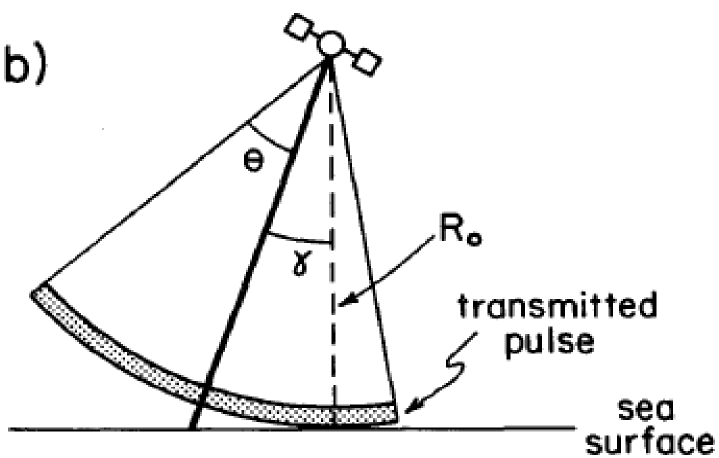
Ensuring a high spatial resolution

Chelton, D. B, Walsh, E. J. and MacArthur, J. L "Pulse compression and sea level tracking in satellite altimetry", J. Atmos. Oceanic Technology, 6, 407 - 438, 1989.



Narrow antenna gain

1. Large antenna size (resolution of 5 km at a wavelength of 2 cm microwave, satellite altitude of 1000 km diameter antenna 4 m)
2. Large uncertainties associated with determining the position of the nadir point (deviation antenna on 0.04o gives an error in determining the height of 20 cm)

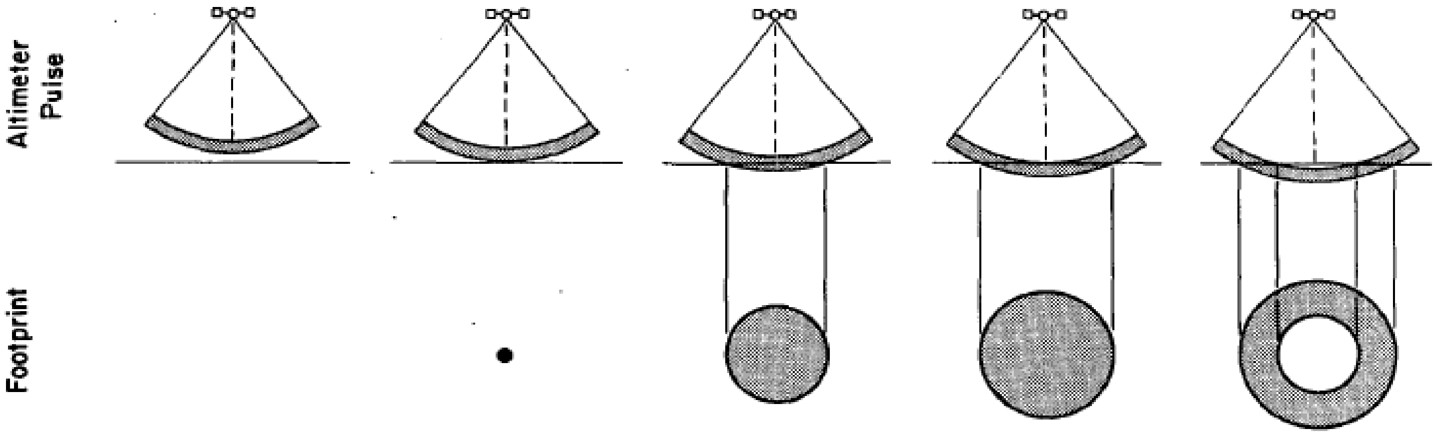


Wide antenna gain the short pulse

1. Moderate antenna size (beam width 2^o , wavelength 2 cm, antenna diameter less than 1m), footprint 50 km
2. There are no uncertainties associated with the determination of the nadir position
3. When the impulse duration τ=3.125 ns and a satellite altitude of 1,300 km maximum resolution of approximately 700 m

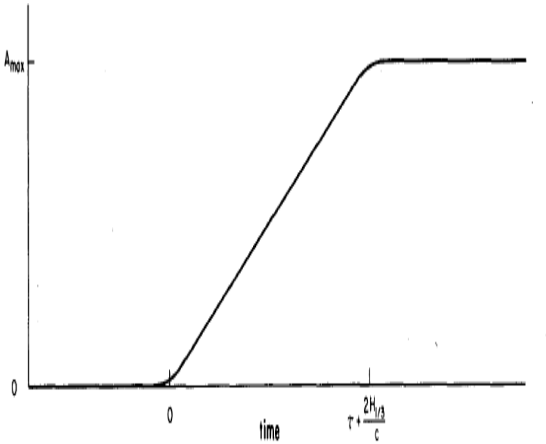
Reflection of a short pulse from a diffuse surface

Chelton, D. B, Walsh, E. J. and MacArthur, J. L "Pulse compression and sea level tracking in satellite altimetry", J. Atmos. Oceanic Technology, 6, 407 - 438, 1989.



The radar altimeter receives the reflected wave (or echo), which varies in intensity over time. Where the sea surface is flat, the reflected wave's amplitude increases sharply from the moment the leading edge of the radar signal strikes the surface.

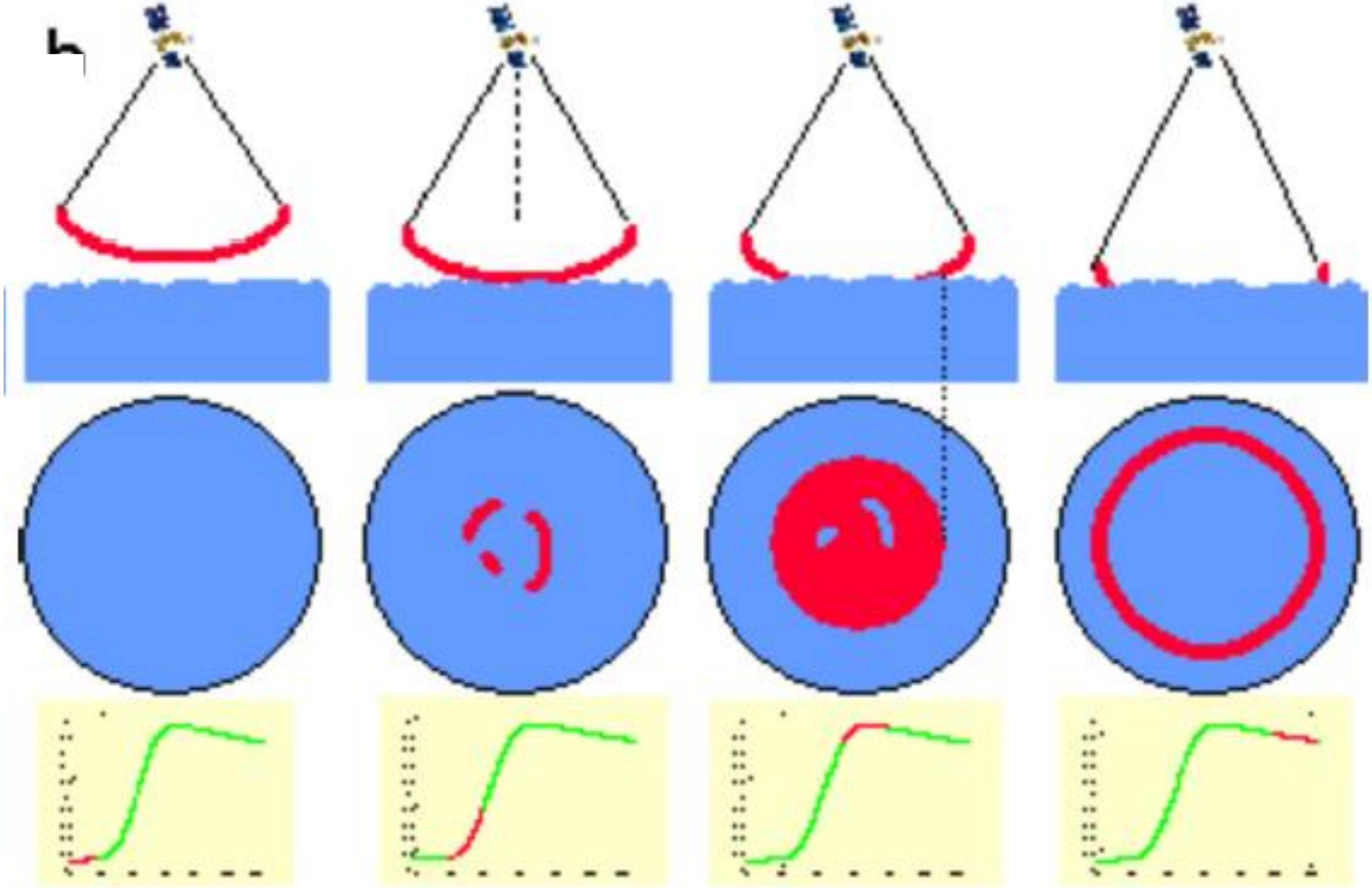
In the scattering of microwaves from a diffuse surface signals coming from different parts of the surface add incoherently (fold capacity) and the received signal is proportional to the illuminated area



The illuminated area increases proportionally to the time

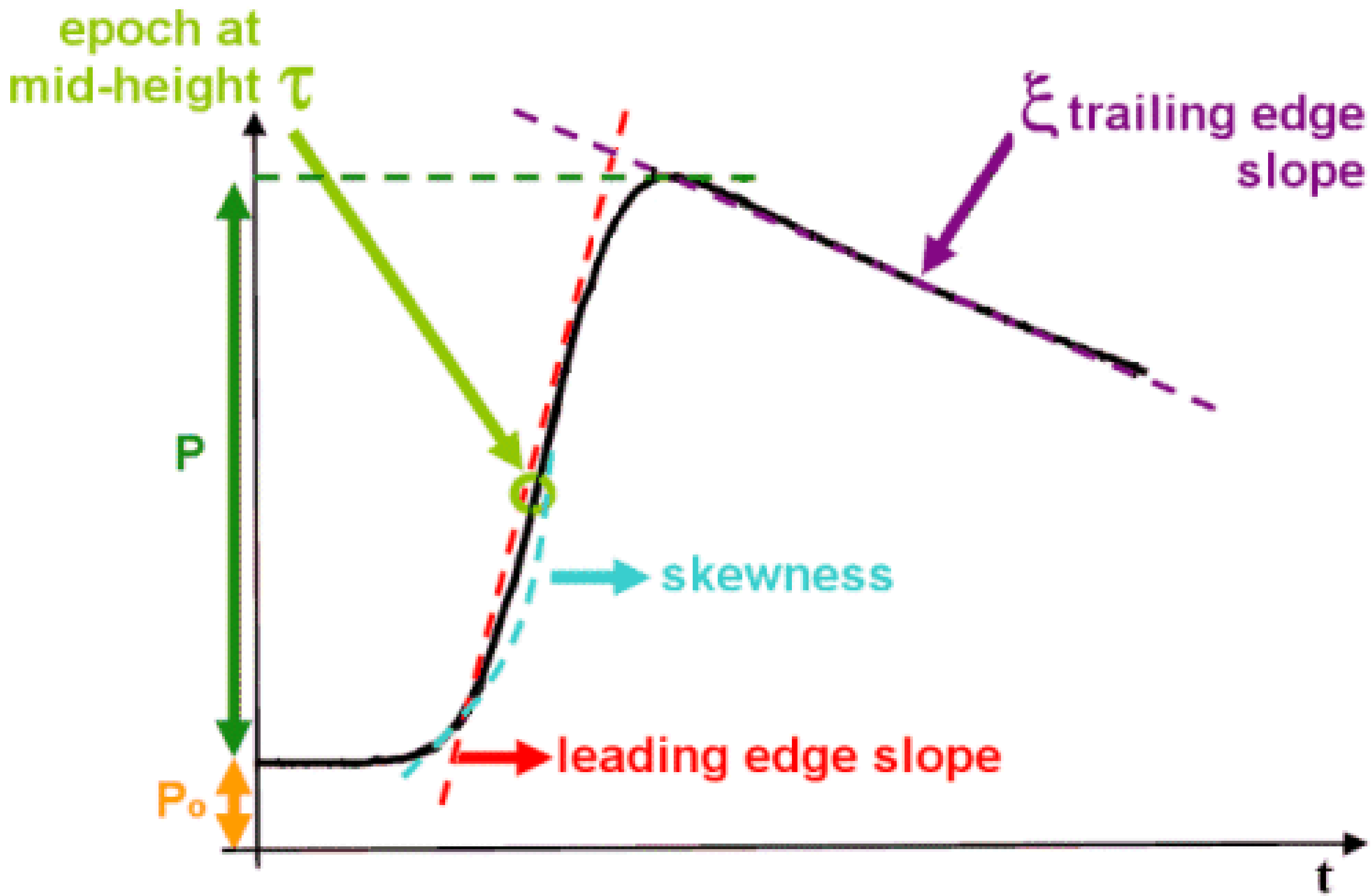
- **distance from the surface is determined by the position of the half-width pulse**
- **spatial resolution - the radius of the illuminated spot**

$$\sqrt{\frac{R_0 \tau c}{1 + R_e / R_0}}$$



In sea swell or rough seas, the wave strikes the crest of one wave and then a series of other crests which cause the reflected wave's amplitude to increase more gradually. We can derive ocean wave height from the information in this reflected wave, since the slope of the curve representing its amplitude over time is proportional to wave height.

WAVEFORMS



The waveform (the shape of the reflected signal) represents the time evolution of the reflected power as the radar pulse hits the surface. In a definite time interval the signal is formed by the reflection from a definite footprint. This time interval is called a telemetric gate.

Trackers

On board the instrument can only measure over a narrow range window (typically 60 m vertically), called the “analysis window.” The purpose of the on-board tracker is to keep the reflected signal from the Earth’s surface within the altimeter analysis window.

Altimeter	Launch - End	H (km)	Inclination	Band	Radar Frequency (GHz)	On-board range tracker
Jason-1	7 Dec 2001–present	1336	66°	Ku	13.6	Split Gate Tracker
				C	5.3	Slaved to Ku
Envisat	1 Mar 2002–present	784	98°	Ku	13.6	Model Free Tracker
				S	3.2	Slaved to Ku
Jason-2	20 Jun 2008–present	1336	66°	Ku	13.6	Median Tracker & DIODE/DEM Tracker
				C	5.3	Slaved to Ku

Re-trackers

- In order to obtain the highest possible accuracy on range measurements over the ocean, today's altimeters downlink the waveforms to Earth and the final retrieval of geophysical parameters from the waveforms is performed on the ground. This is called "waveform re-tracking".

- The aim of the ground-based re-tracking is to fit a model or functional form to the measured waveforms, and retrieve geophysical parameters such as the range, echo power, etc.

- Functional forms can be purely empirical, or, as in the case of the Brown ocean retracker, be based on physics.

Empirical Retrackers

Offset Centre of Gravity Retracker (OCOG)

Based on the definition of a rectangle about the effective centre of gravity of the waveform, the amplitude (A) and width (W) of the waveforms and the gate position of the waveform centre of gravity (COG) are estimated from the waveform data using:

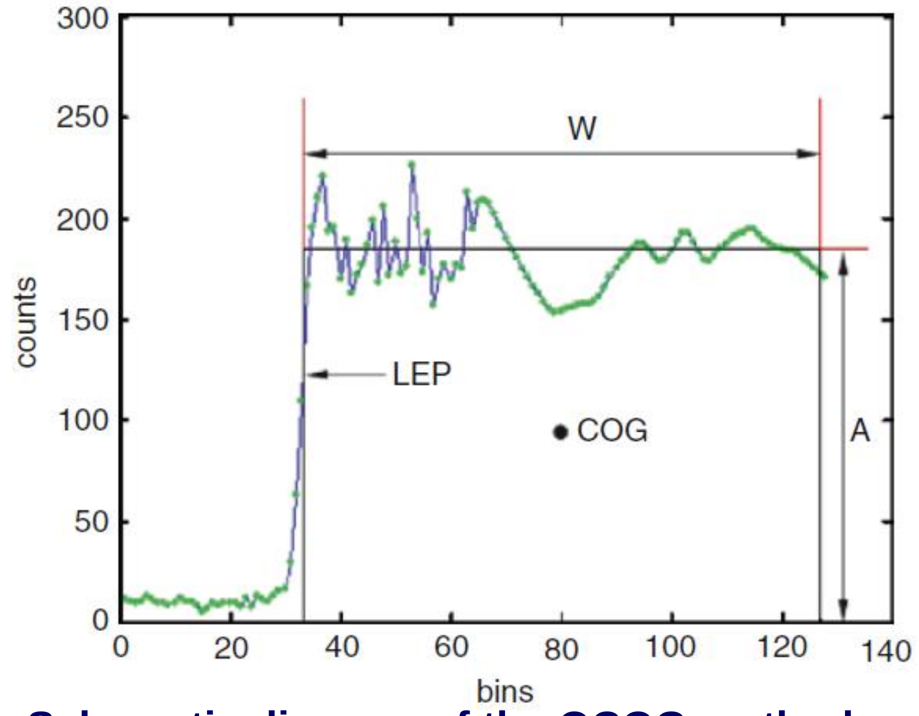
$$A = \sqrt{\sum_{i=1+n_1}^{N-n_2} P_i^4(t) \Big/ \sum_{i=1+n_1}^{N-n_2} P_i^2(t)}$$

$$W = \left(\sum_{i=1+n_1}^{N-n_2} P_i^2(t) \right)^2 \Big/ \sum_{i=1+n_1}^{N-n_2} P_i^4(t)$$

$$COG = \sum_{i=1+n_1}^{N-n_2} i P_i^2(t) \Big/ \sum_{i=1+n_1}^{N-n_2} P_i^2(t)$$

The leading edge position (LEP) is given by:

$$LEP = COG - \frac{W}{2}$$



Schematic diagram of the OCOG method

Ice-1 retracker for the Envisat RA-2 altimeter

Threshold Retracker

Calculate the thermal noise:

$$P_N = \frac{1}{5}\sum_{i}^{5} p_i$$

Compute the threshold level:

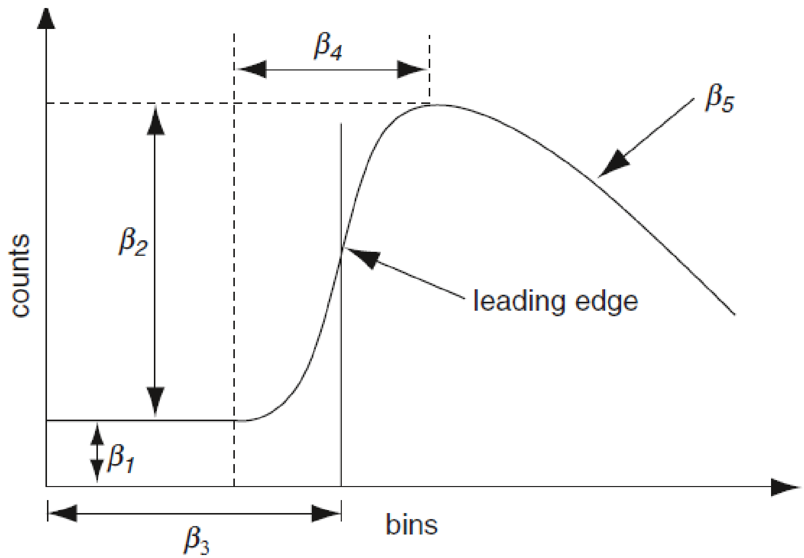
$$T_h = P_N + q \cdot (A - P_N)$$

The retracked range on the leading edge of the waveform is computed by linearly interpolating between the bins adjacent to *Th* using

$$G_r = G_{k-1} + \frac{T_h - P_{k-1}}{P_k - P_{k-1}}$$

A* is determined by Offset Centre of Gravity Retracker, P_N is the averaged value of the power in the first five gates, *q* is the threshold value (e.g., 50%), *Gk* is the power at the kth gate, where k is the location of the first gate exceeding *Th

The β-parameter Retracker



Schematic diagram of 5-parameter β-retracker waveform

The general expression for the 5-parameter functional form of the returned power y(t)

$$y(t) = \beta_1 + \beta_2(1 + \beta_5 Q) P\left(\frac{t - \beta_3}{\beta_4}\right)$$

$$Q = \begin{cases} 0 & for \quad t < \beta_3 + 0.5\beta_4 \\ t - (\beta_3 + 0.5\beta_4) & for \quad t \geq \beta_3 + 0.5\beta_4 \end{cases}$$

$$P(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-q^2}{2}\right) dq$$

The unknown parameters β relate to the shape of the waveform as shown in Fig. and as follows: β1 is the thermal noise level, β2 the return signal amplitude, β3 the mid-point on the leading edge, β4 the waveform rise-time, and β5 the slope of the trailing edge.

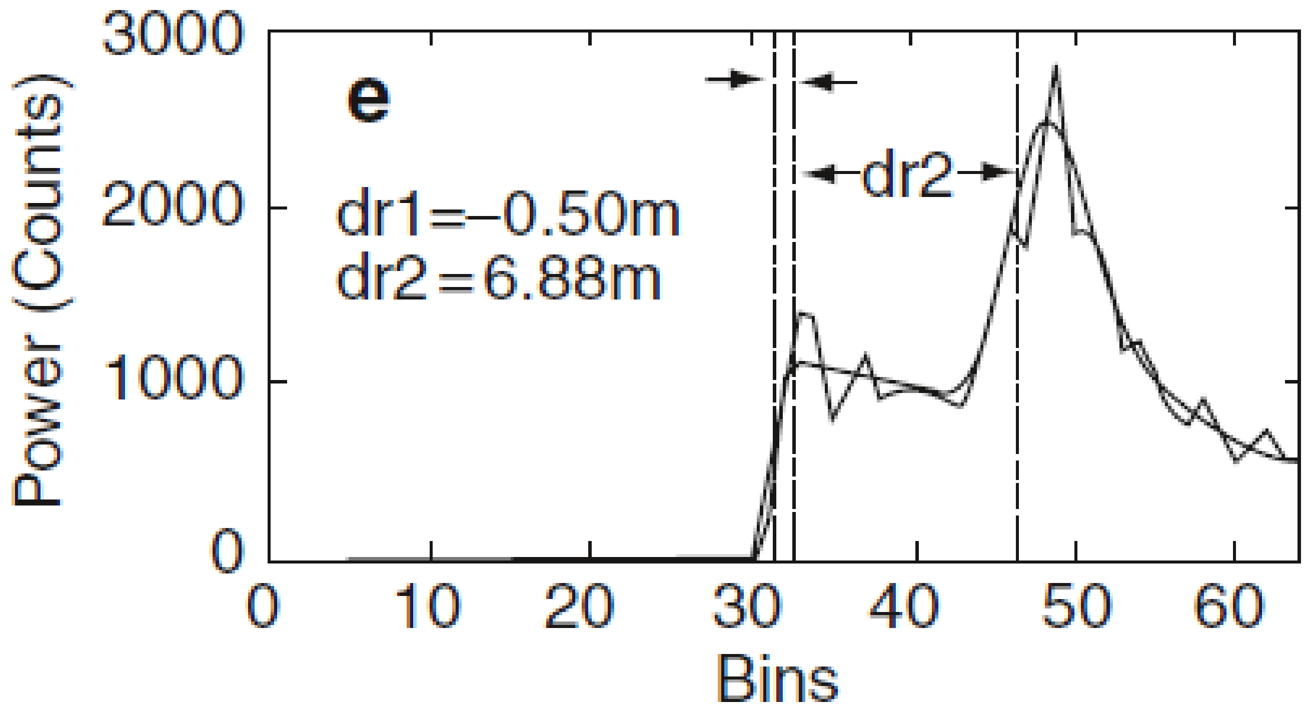
The waveforms created with the empirical b-parameter function show marked similarities with the waveforms produced with the theoretical Brown ocean waveform model. However, the empirical β parameters remain simple fitted parameters, not related to physical properties.

The β-parameter Retracker

The 9-parameter form of the β-retracker with exponential trailing edge reads

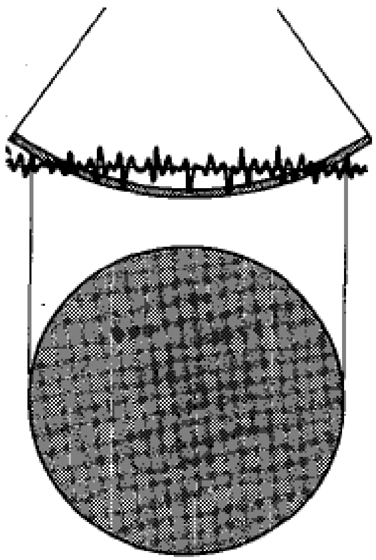
$$y(t) = \beta_1 + \sum_{i=1}^{2} \beta_{2i} \exp(-\beta_{5i} Q_i) P\left(\frac{t - \beta_{3i}}{\beta_{4i}}\right)$$

$$Q_i = \begin{cases} 0 & for \quad t < \beta_{3i} - 2\beta_{4i} \\ t - (\beta_{3i} + 0.5\beta_{4i}) & for \quad t \geq \beta_{3i} - 2\beta_{4i} \end{cases}$$



Physically-based Retrackers

THE AVERAGE ROUGH SURFACE IMPULSE RESPONSE



$$P_i(t) = \int_{-\infty}^{\infty} q\left(\frac{c}{2}(t - t_1)\right) dt_1 \int_{-\infty}^{\infty} P_{FS}(t_2)\, p(t_1 - t_2)\, dt_2$$

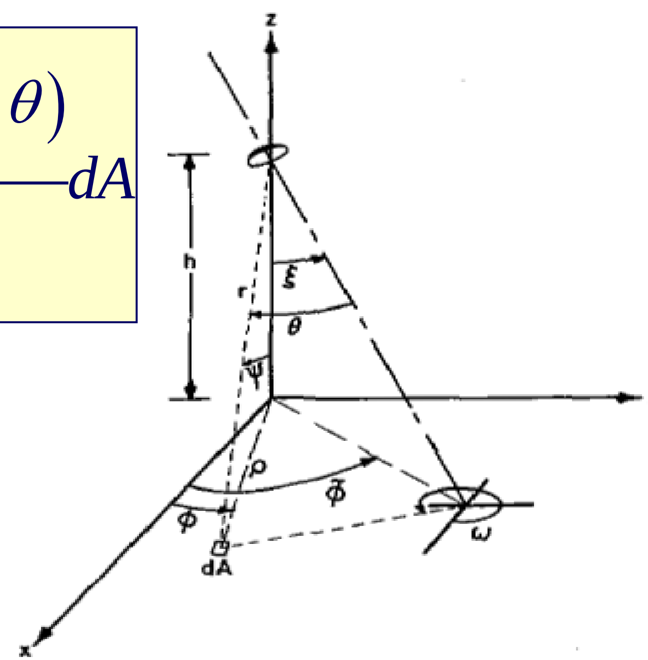
***p(t)* – radar system point target response;**

P_{FS} – the average backscattered power from a mean flat surface;

***q (z)* – probability density of the specular points σ;**

Brown, G. S. (1977), The average impulse response of a rough surface and its applications, IEEE Trans. Antennas Propag., AP-25, 67–74.

$$P_{FS}(t) = P_0 \iint\limits_{\substack{illuminated\\ area}} \frac{\delta\left(t - \frac{2r}{c}\right) G^2(\theta)\,\sigma(x, y, \theta)}{r^4}\, dA$$



G - gain of the radar antenna

***r* – range from the radar to the elemental scattering area dA on the surface ,**

σ - backscattering cross section per unit scattering area

***h* – the mean distance from the satellite**

Suppositions :

Small-angle approximation valid for satellite altimetry

$$r=\sqrt{h^2+x^2+y^2}\approx h+\rho^2/2h;\ \rho^2=x^2+y^2$$

Small off-nadir angle

Gaussian probability density of specular points, s=SWH/2

$$q(z)=\frac{1}{s\sqrt{2\pi}}\exp(-z^2/2s^2)$$

Model expressions for backscattering cross section per unit scattering area, gain of the radar antenna and initial impulse shape

$$\sigma=\sigma^{(0)}\cdot e^{-tg^2\theta/\alpha}$$

$$\mathrm{G}(\theta)=\exp(-2\sin^2\theta/\gamma)$$

$$\theta^2=\rho^2/h^2$$

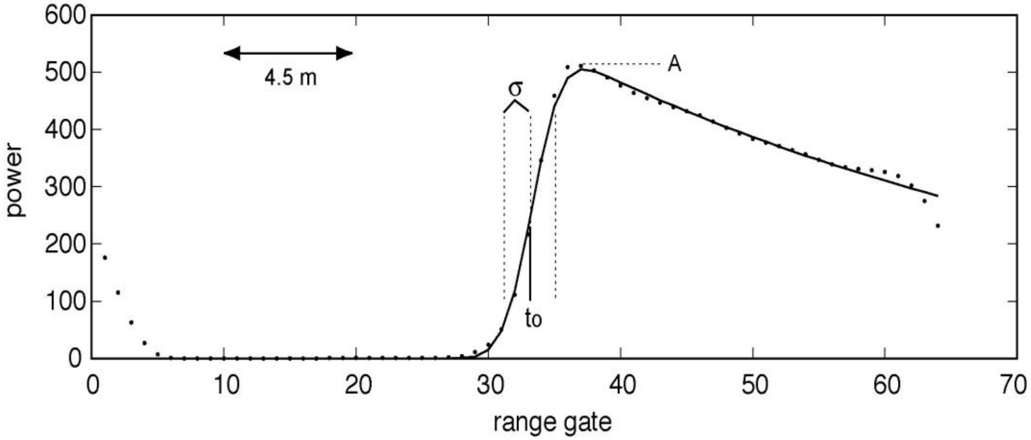
$$p(t)=\frac{1}{\tau\sqrt{2\pi}}\exp(-t^2/2\tau^2)$$

The Brown formula

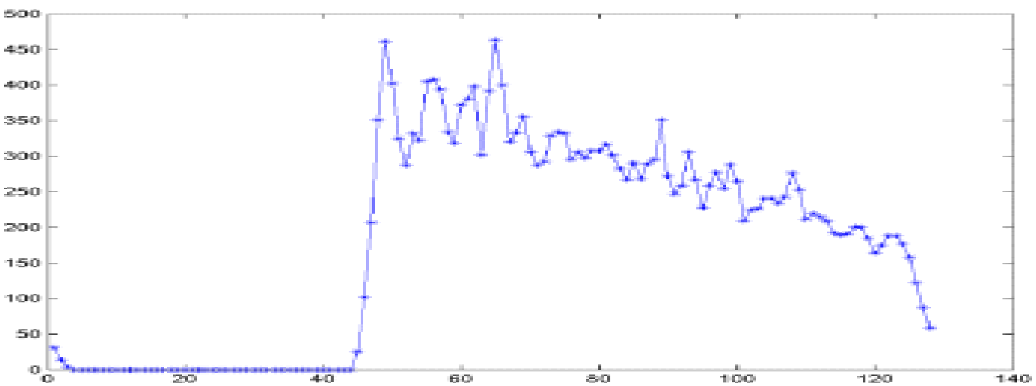
$$P\left(t-\frac{2h}{c}\right)=\frac{P_0\sigma^{(0)}}{2h^4}e^{-\left(\frac{4}{\gamma}+\alpha\right)\frac{(ct-2h)}{h}}\left(1+\mathrm{erf}\left(\frac{(ct-2h)}{\sqrt{2}\sqrt{s^2+c^2\tau^2}}\right)\right)\times$$
$$\times\exp\left[-\frac{4}{\gamma}\sin^2\xi-\frac{c}{h}\left(t-\frac{2h}{c}\right)\left(\frac{4}{\gamma}\cos 2\xi+\alpha\right)\right]I_0\left(\frac{4}{\gamma}\sin 2\xi\sqrt{\frac{c}{h}\left(t-\frac{2h}{c}\right)}\right)$$

Open ocean

Waveform according to Brown's formula



Waveform in the open ocean



Approximation of the waveform by Brown's formula (Ocean-1,2 re-tracking)

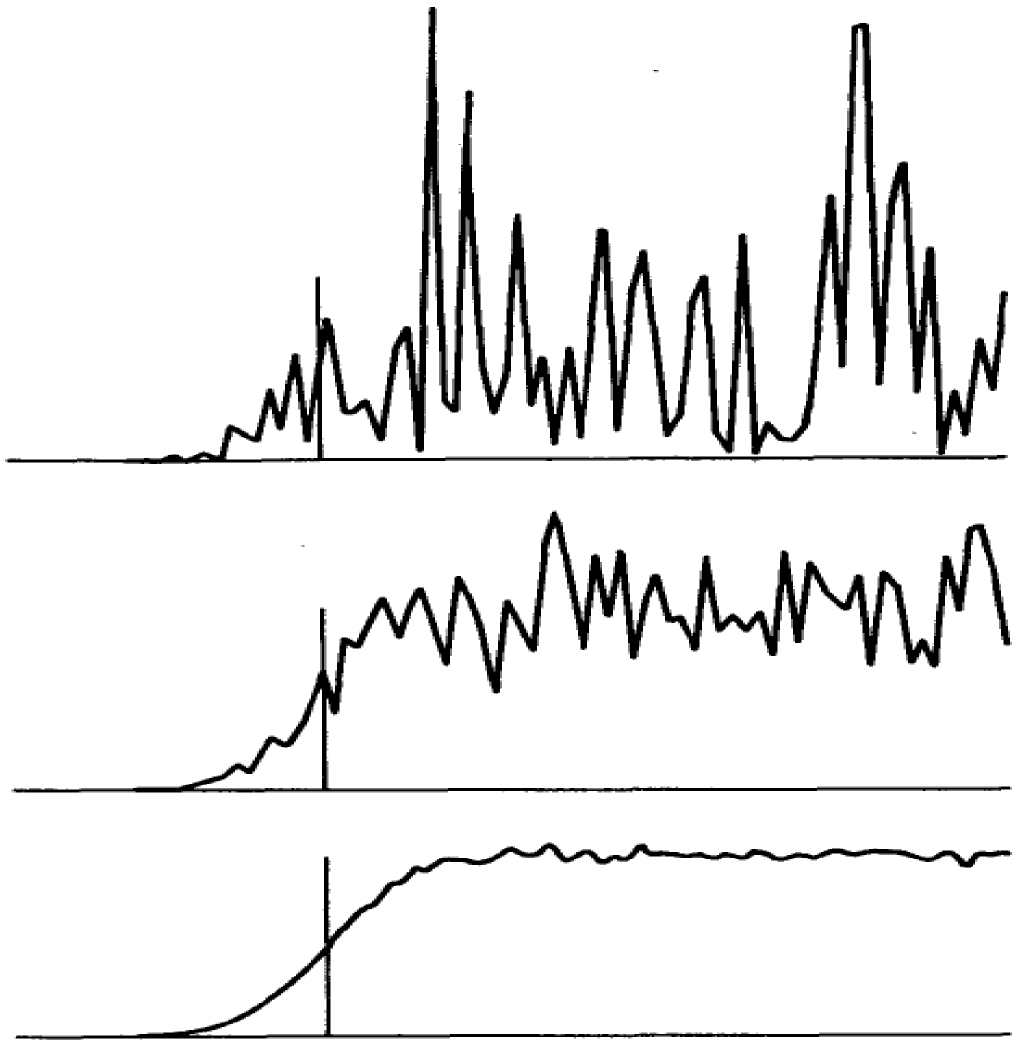
$t_0=2h/c$– epoch at mid-height: this gives the time delay of the expected return of the radar pulse (estimated by the tracker algorithm) and thus the time the radar pulse took to travel the satellite-surface distance (or 'range') and back again

***SWH*=*2s* – leading edge slope: this can be related to the significant wave height (SWH)**

***A* – the amplitude of the useful signal. This amplitude with respect to the emission amplitude gives the backscatter coefficient, sigma0– roughness parameter related to the wind speed**

FIG. 12. Averages of 1, 25, and 1000 (top to bottom) simulated altimeter waveforms with realistic Rayleigh-distributed geophysical noise. (After Townsend et al. 1981.)

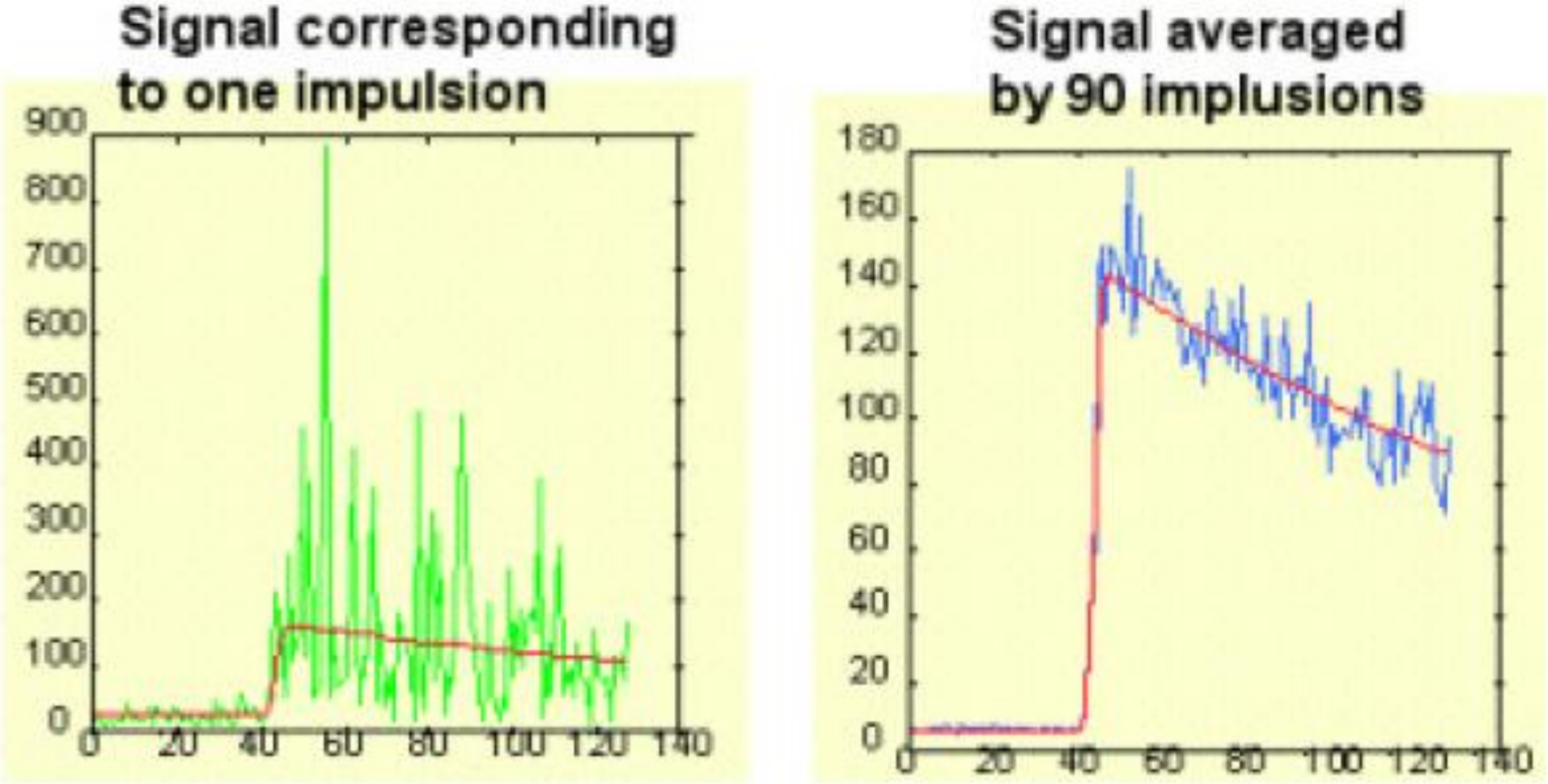
Chelton, D. B, Walsh, E. J. and MacArthur, J. L "Pulse compression and sea level tracking in satellite altimetry", J. Atmos. Oceanic Technology, 6, 407 - 438, 1989.



1 waveform

Average of 25 waveforms

Average of 1000 waveforms



In practice, in sea swell or land, the illuminated surface is never flat. This effect is translated into a distortion and high frequency noise in the waveforms. To increase the signal/noise ratio, the individual echoes are averaged by 50 (ERS-1&2) or by 90 (Topex/Poseidon -Poseidon-1 altimeter- and Jason-1), making the elementary waveforms that will be transmitted to the ground.

Operating characteristics of altimeters

Altimeter	Band	Antenna Beamwidth (deg)	PRF (Hz)	Number of waveform gates	Nominal tracking point	Gate width (ns)	Waveform averaging frequency (Hz)	Number of waveforms averaged
Geosat	Ku	2.0°	1020	60	30.5	3.125	10	100
ERS-1	Ku	1.3°	1020	64	32.5	3.03	20	50
ERS-2	Ku	1.3°	1020	64	32.5	3.03	20	50
TOPEX	Ku	1.1°	4500	128	32.5	3.125	10	456
	C	2.7°	1200	128	35.5	3.125	5	240
Poseidon	Ku	1.1°	1700	60	29.5	3.125	20	86
GFO	Ku	1.6°	1020	128	32.5	3.125	10	100
Jason-1	Ku	1.28°	1800	104*	31	3.125	20	90
	C	3.4°	300	104*	31	3.125	20	15
Envisat	Ku	1.29°	1800	128	46.5	3.125	18	100
	S	5.5°	450	64	25.5	6.25	18	25
Jason-2	Ku	1.26°	1800	104*	31	3.125	20	90
	C	3.38°	300	104*	31	3.125	20	15

Databases of satellite altimetry Jason-1,2

GDR (Geophysical Data Records)

Contents: along-track altimetric measurements averaged over 1 second, corrections to apply in delayed time (GDR) or near-real time (IGDR) (data on sea level altitude, wind speed, excitement, supporting information)

Use: geophysical studies, operational oceanography (near-real time)

Sensor Geophysical products

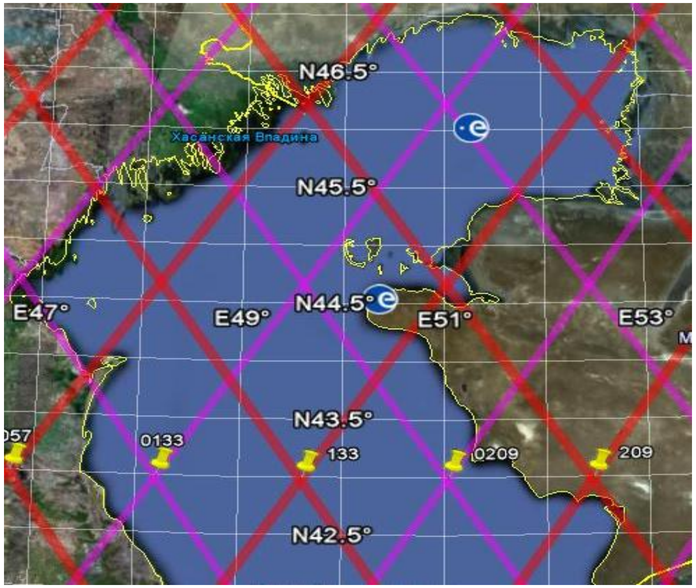
Contents : Data of GDR + 20 Hz along-track waveform information, corrections to apply.

Use: expert use; coastal, ice studies or anything requesting a different retracking function than the one used for ocean

ftp://avisoftp.cnes.fr/AVISO/pub/

Using satellite altimetry in the coastal zone, on land and inland waters

Ground tracks satellites Jason-1, 2 in the waters of some inland waters



Northern
Caspian Sea

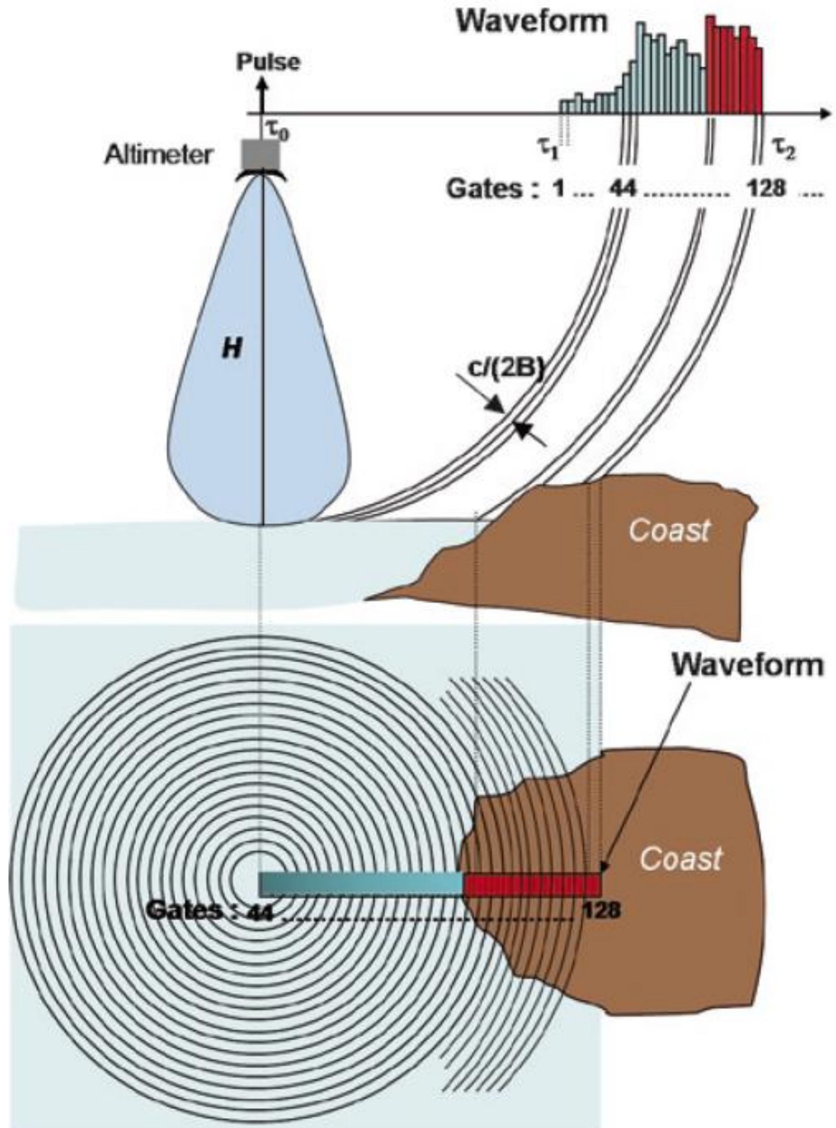
Gorky
reservoir

Rybinsk
reservoir

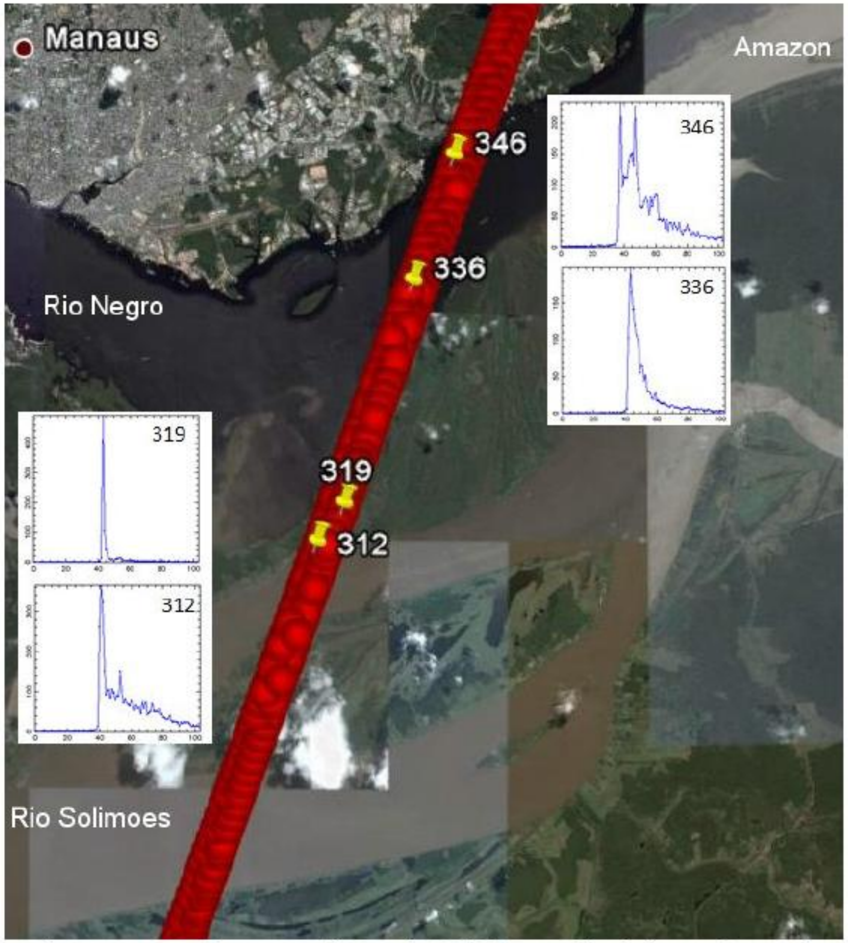
Red ones – Jason-2, Jason-1 (before maneuver in June 2009)

Magenta ones - Jason-1 (after maneuver in June 2009)

The power received in a given gate are linked to the relative proportion of sea and land area in the corresponding footprint, and to the reflective properties (sigma_0, the backscatter coefficients) of each type of surface. Fig. shows how and when the nadir of the satellite is still over ocean, it is the last samples in the waveform that are contaminated first by land returns. The number of contaminated samples will depend on the height and surface area of the land, as well as its proximity to the nadir point.



Gommenginger C., Thibaut P, Fenoglio-Marc L, Quartly G, Deng X, Gomez-Enri J, Challenor P, and Gao Y, « Retracking altimeter waveforms near the coasts ; A review of retracking methods and some applications to coastal waveforms”, DOI: 10.1007/978-3-642-12796-0_6, Springer-Verlag Berlin Heidelberg 2010.



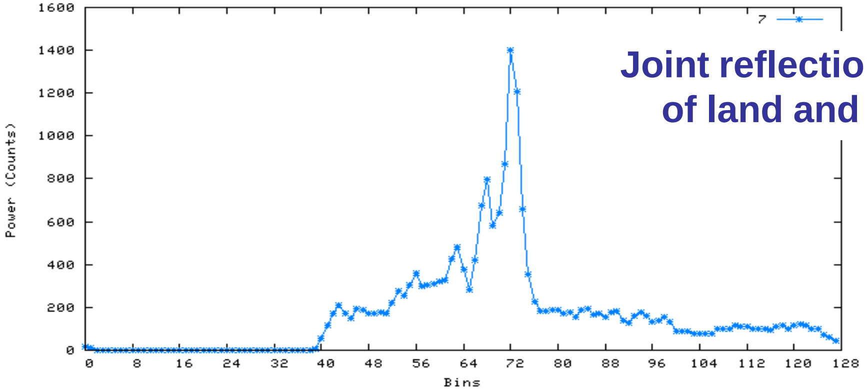
Jason-2 waveforms on Amazon river : specular waveform (n°319) result of the return signal from very reflective surfaces like water bodies, multipeaked waveforms result of heterogeneous targets in the footprint

The continental lands are composed of a very varying heterogeneous surfaces which reflect as much of varying waveforms. The altimeter footprint is frequently composed of and contaminated by a multiplicity of surfaces.
Thus, waveforms on these surfaces include a wide variety of configurations which are difficult to classify and process.
Altimeter data over land must be post-processed in an another way that the Brown model

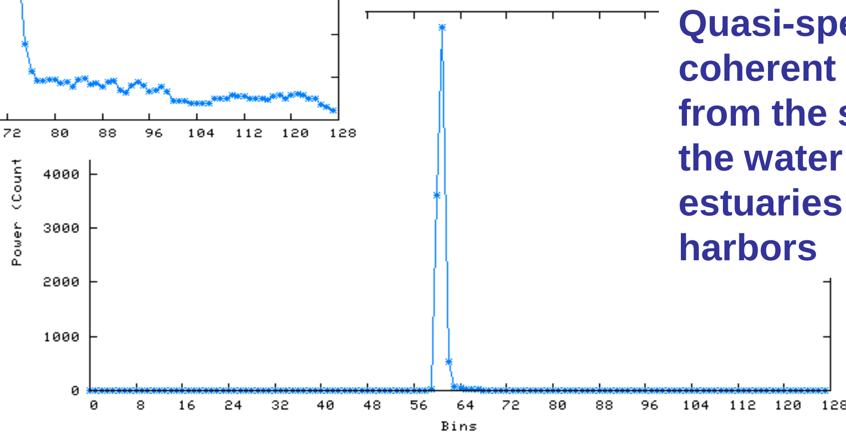
Causes of error standard algorithm re-tracking

incorrect determination of the position of the leading edge of the reflected impulse

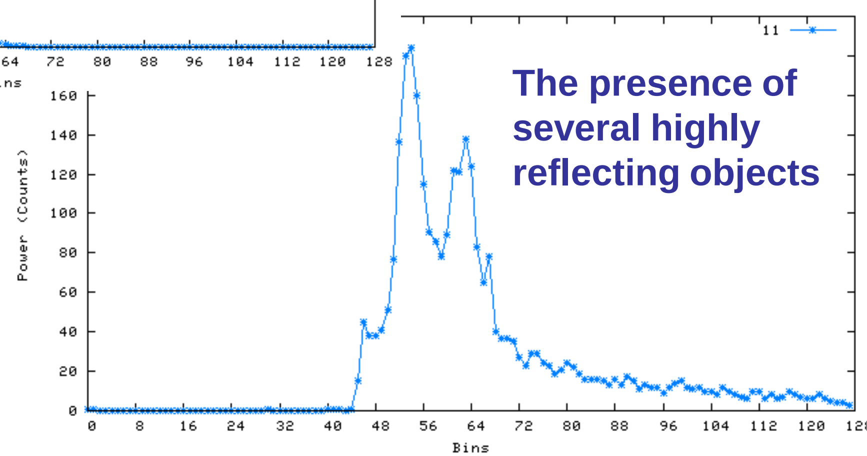
Waveforms in the coastal area and inland waters



Joint reflection of land and water



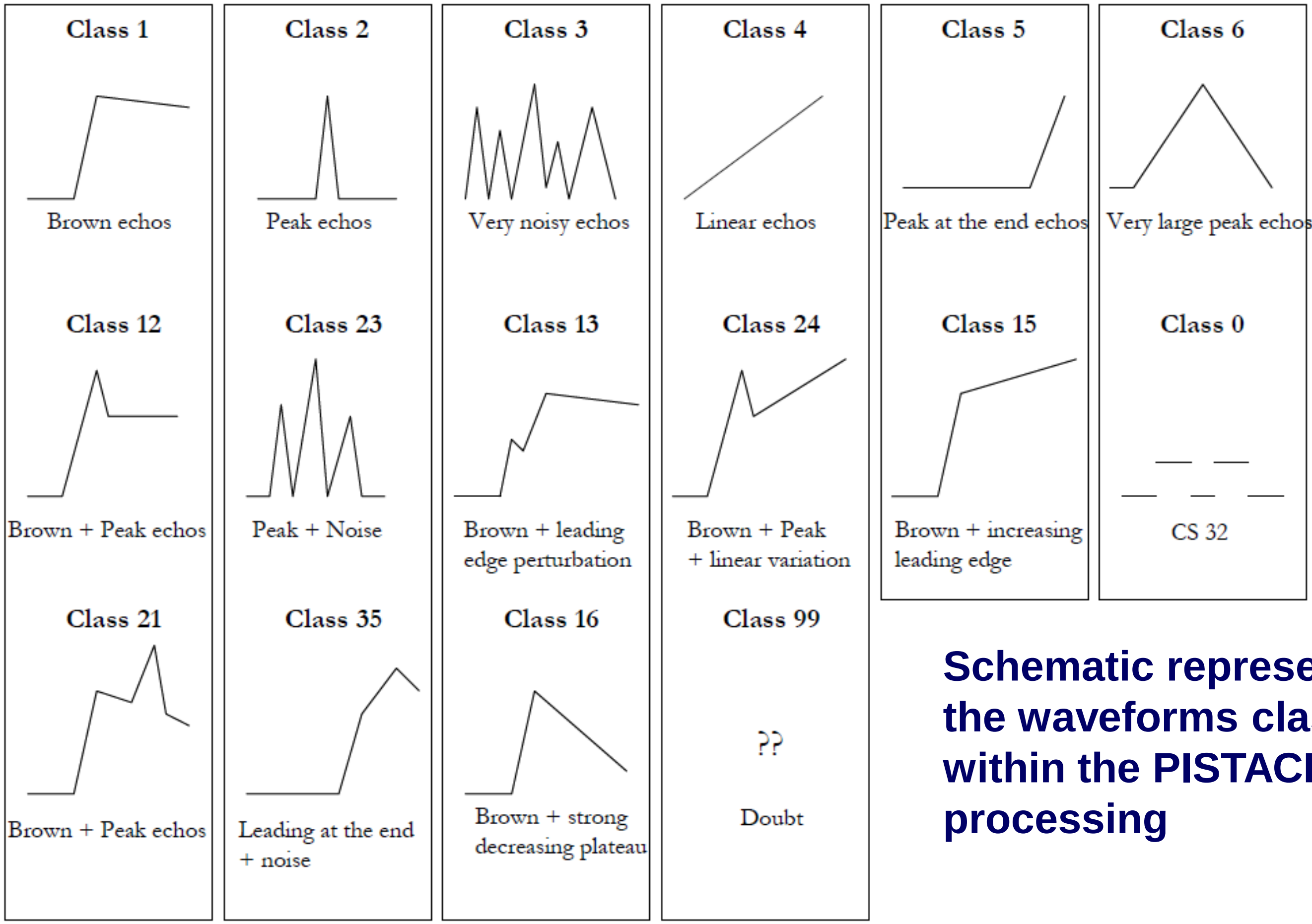
Quasi-specular, coherent reflection from the surface of the water in estuaries and harbors



The presence of several highly reflecting objects

PISTACH Processing and Outputs for Coastal and Continental Water Areas

A classification of waveforms has been achieved (neural network algorithm).



Schematic representation of the waveforms classes within the PISTACH processing

Knowing, for each waveform the classification number (see Figure 4) can also be very useful to choose which retracking output has to be preferably considered

PISTACH Processing and Outputs for Coastal and Continental Water Areas

Retracking strategies

- **Ice1** retracking is based on the Offset Centre of Gravity (OCOG)

- **Ice3** : The ice3 retracker is deduced from the ice1 retracker. Its principle is exactly the same than the ice1 one except that computations are done in a smaller window selected around the main leading edge of the waveform [-10;+20 samples].

- **Oce3** :This algorithm is an classical MLE3 retracking algorithm

For these retracking algorithms, the 20Hz retracking ouput (ranges, sigma0, SWH, classes etc.) are provided in the Pistach products.

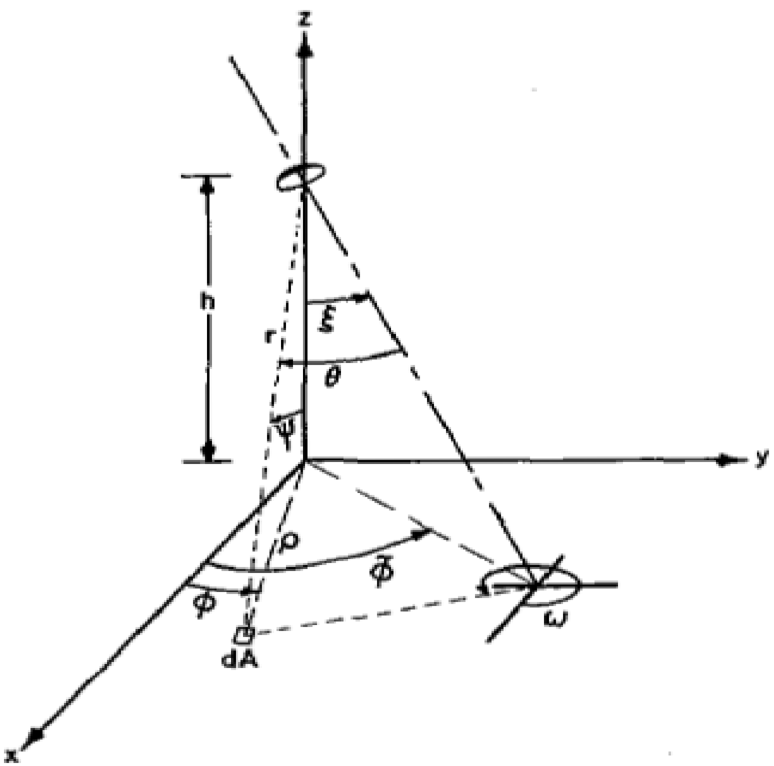
Theoretical model of waveforms

Based on the theory of non-coherent scattering of microwaves by rough surface

Geometry (Brown, 1977)

The average impulse response of the rough surface

$$P_i(t) = P_0 \iint\limits_{\substack{illuminated \\ area}} \frac{G^2(\theta)\sigma(x,y,\theta)}{r^4} dA \int_{-\infty}^{\infty} p\left(t_1 - \frac{2r}{c}\right) q\left(x, y, \frac{c}{2}(t - t_1)\right) dt_1$$



G - gain of the radar antenna
r - range from the radar to the elemental scattering area *dA* on the surface
h - the mean distance from the satellite
σ- backscattering cross section per unit scattering area
q(z) - height probability density of specular points

Suppositions

1. Small-angle approximation valid for satellite altimetry

$$r=\sqrt{\left(h+H\left(x,y\right)\right)^{2}+x^{2}+y^{2}}\approx\left(h+H\left(x,y\right)\right)+\rho^{2}/2h^{2};\ \rho^{2}=x^{2}+y^{2}$$

2. Zero off-nadir angle
3. Gaussian probability density of specular points

$$q(z)=\frac{1}{s(x,y)\sqrt{2\pi}}\exp(-z^{2}/2s(\rho,\varphi)^{2})$$

4. Model expressions for backscattering cross section per unit scattering area, gain of the radar antenna and initial impulse shape

$$\sigma=\sigma^{(0)}\left(\rho,\varphi\right)\cdot e^{-tg^{2}\theta/\alpha}$$

$$\mathrm{G}(\theta)=\exp(-2\sin^{2}\theta/\gamma)$$

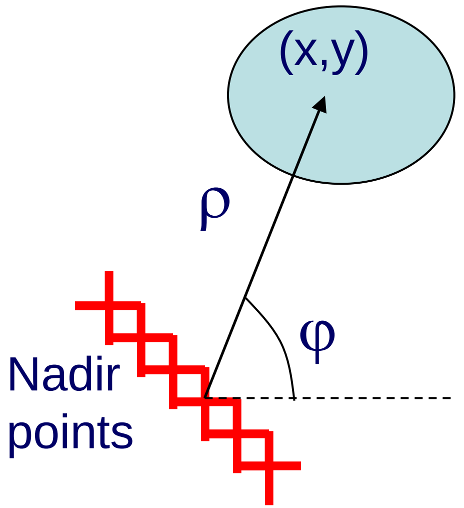
$$\theta^{2}=\rho^{2}/h^{2}$$

$$p(z)=\frac{1}{\tau_{i}\sqrt{2\pi}}\exp(-z^{2}/2\tau_{i}^{2})$$

The average impulse response of the rough inhomogeneous surface

$$P_i(\tau)=\frac{P_0}{\sqrt{2\pi}h^4}\int_0^{\infty}\int_0^{2\pi}\frac{\sigma^{(0)}(\rho,\varphi)}{\sqrt{s^2(\rho,\varphi)+c^2\tau_i^2}}e^{-\left(\frac{4}{\gamma}+\alpha(\rho,\varphi)\right)\frac{\rho^2}{h^2}}\times$$

$$\times\exp\left\{-\frac{\left(c\tau-2H(\rho,\varphi)-\rho^2/h\right)^2}{8\left(s^2(\rho,\varphi)+c^2\tau_i^2\right)}\right\}\rho d\rho d\varphi$$



Parameters in the formula are the functions of the coordinates of the surface.

<u>For water surface</u>

Elevation *H* is the water level

s is significant wave height

σ is determined by the wind speed

<u>For land surface</u>

Elevation *H* is determined by topography

s is surface roughness

σ is determined by the reflecting properties of the surface

s and σ depend on ρ and φ

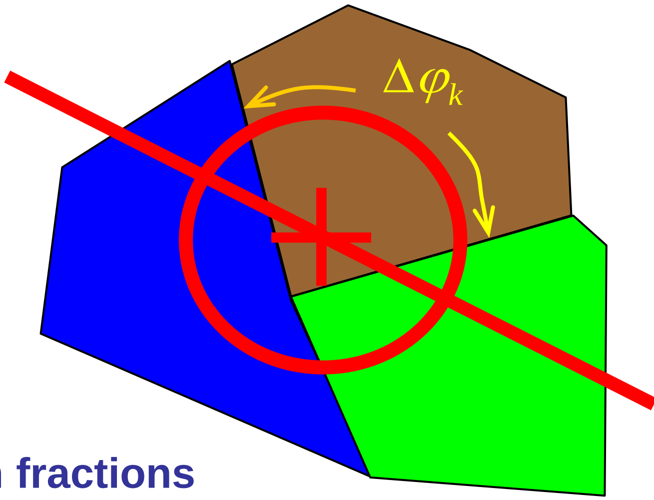
Along-track topography and parameters of the surface are required to construct the model

Step-by-step strategy of local adaptive re-tracking algorithms for retrieving water level for complex area (land, coastal zone, inland waters, etc) from satellite altimetry

1. Local geographical model of reflecting surface
2. Solving the direct problem, modeling waveforms within the model
3. Imposing restrictions and validity criteria for the algorithm basing on waveform modeling
4. Solving the inverse problem
    1. First step – retrieving a tracking point by threshold algorithm
    2. Second step – refinement of the tracking point and estimating SWH, σ_0 by solving the optimization problem

Piecewise constant geographical model for complex area (land, coastal zone, inland waters, etc) from satellite altimetry

Fractions of the surface with constant parameters



Average impulse response

$$P_i(\tau) = P(\tau) + P_{specula}(\tau)$$

Contribution of the surface is the sum of contributions from fractions

$$P(\tau) = P_0 \sum_{k=1}^{N} \sigma_k^{(0)} e^{-\left(\frac{4}{\gamma}+\alpha_k\right)\frac{(c\tau-2H_k)}{h}} \left(1 + \mathrm{erf}\left(\frac{(c\tau - 2H_k)}{2\sqrt{2}\sqrt{s_k^2 + +c^2\tau_i^2}}\right)\right) \times$$

$$\times \Delta\varphi_k\left(x_N, y_N, \sqrt{h(c\tau - 2H_k)}\right)$$

Known parameters are H_k , topography of the land

Parameters for fitting $H_{water}, \sigma_k^{(0)}, s_k, \alpha_k$

The first step is re-tracking by the threshold criterion, the second step is refinement by fitting the parameters.

SWH and σ_0 also can be retrieved.

Step-by-step strategy of local adaptive re-tracking algorithms for retrieving water level for complex area (land, coastal zone, inland waters, etc) from satellite altimetry

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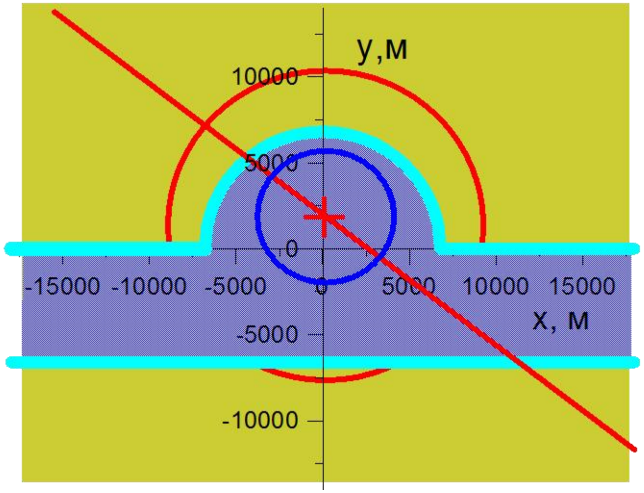
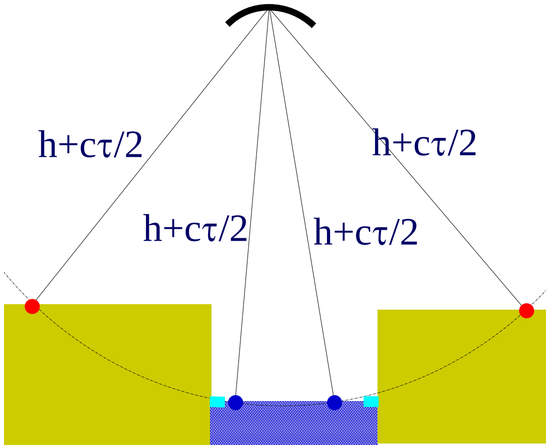
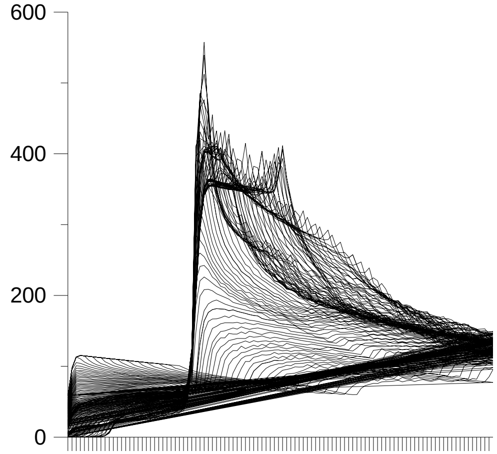
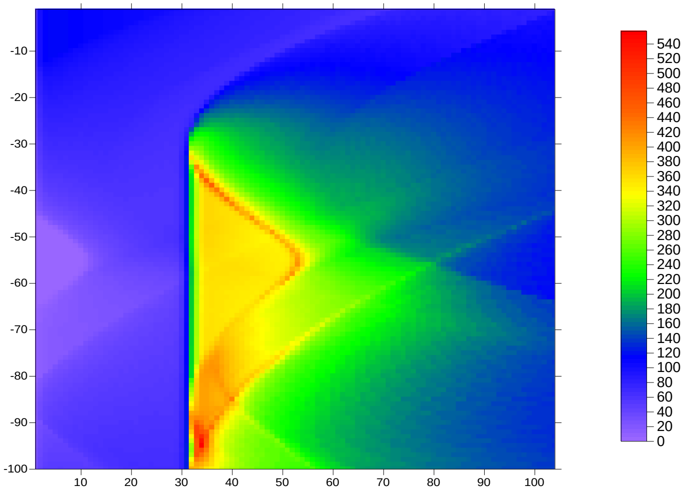
An example of piecewise constant geographical model of the Gorky reservoir





Modeled waveforms





Step-by-step strategy of local adaptive re-tracking algorithms for retrieving water level for complex area (land, coastal zone, inland waters, etc) from satellite altimetry

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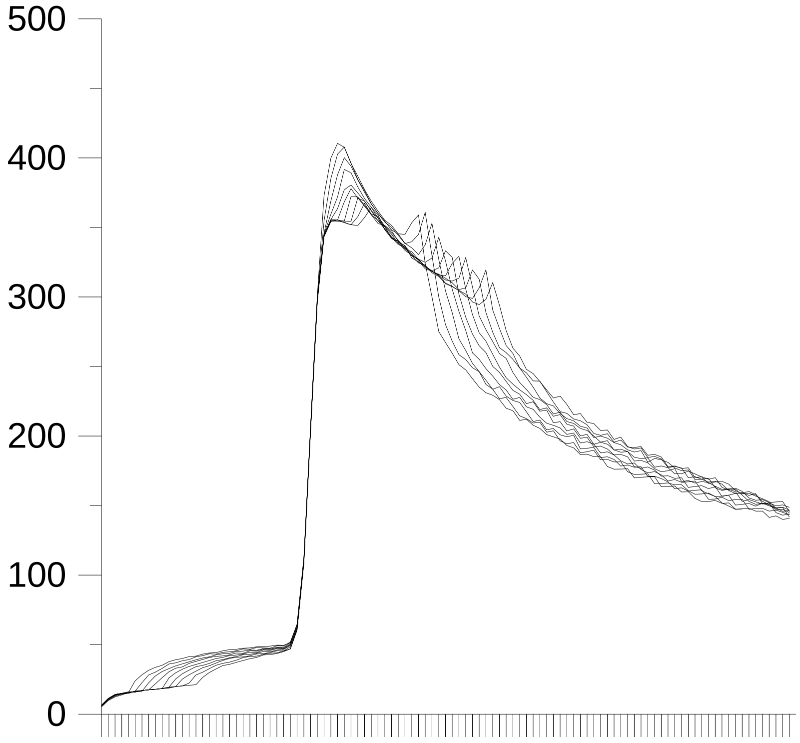
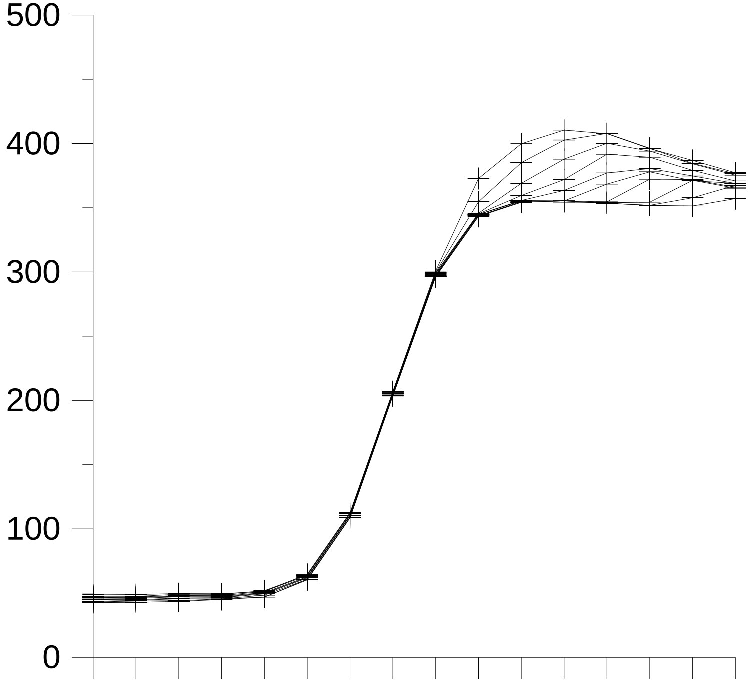
Waveforms with the nadir points at the water

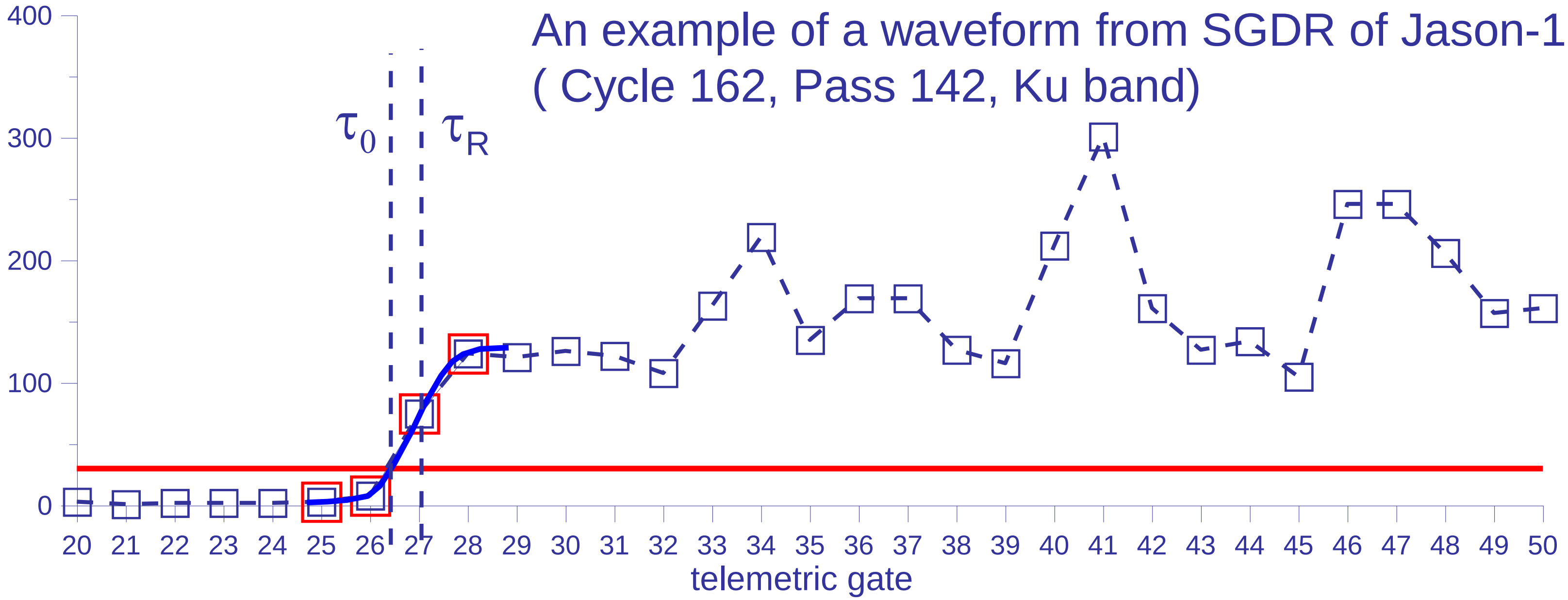


Error function shape of the front of the waveform



The local re-tracking algorithm appropriate for the Gorky reservoir and surroundings for pass 142

1. Threshold re-tracking algorithm, provides tracking point τ_0 , determined by a definite threshold



2. Improved threshold re-tracking algorithm. 4 points in the vicinity of the threshold are fitted by the error function

$$A\left(1+\mathrm{erf}\left(\frac{\tau-\tau_R}{S}\right)\right)$$

The parameters A, τ_R, S are retrieved from an optimization algorithm

Conclusion

- High accuracy of the satellite radar altimetry is achieved because of use of strong a-priory information about the waveforms
- Over the ocean Brown's formula is obtained for a wave form due to several suppositions. The main one is homogeneity of the ocean surface within the footprint. Only in this case re-tracking based on Brown's formula is confirmed
- In case of in-homogeneous backscattering properties of the Earth surface the waveform is a constituent of signals scattered by different parts of the footprint. Then the Brown's formula is of limited applicability.
- Special models of returned impulses are required to describe reflection from the land and inland waters
- We considered several types of re-trackers:
- The adaptive re-tracking algorithms for retrieving water level for complex area (land, coastal zone, inland waters, etc) from satellite altimetry is a universal approach to measuring water level in medium-size land water bodies